Preparation notes, add-on to last years notes / 25/3/2020

Notes on Sync

$$p(\underline{r} \mid \mathcal{H}_1) = \frac{1}{(\pi \sigma_n^2)^{N_p}} \exp\left(\frac{-\|\underline{r} - h p\|^2}{\sigma_n^2}\right)$$

Estimating $\hat{h}$ with ML principle

$$\hat{h} = \arg\max_h p(\underline{r} \mid \mathcal{H}_1, h) \rightarrow \text{maximize exponential}$$

$\rightarrow$ minimize $\|r - hp\|^2$

$$\arg\min_h \underbrace{\|r - hp\|^2}_{L} \qquad L = \cancel{\|r\|^2} - r^H p h - h p^H r + |h|^2 \underbrace{\|p\|^2}_{N_p}$$

write $h = |h| e^{j\phi_h}$

$$L = \underbrace{- r^H p |h| e^{j\phi_h} - p^H r |h| e^{-j\phi_h}}_{-2\,\mathrm{Re}\{p^H r |h| e^{-j\phi_h}\}} + |h|^2 N_p$$

to minimize make Real $\Rightarrow \hat{\phi}_h = \arg(p^H r)$

Substituting $\hat{\phi}_h$ for ϕ yields

$$L = |h|^2 N_p - 2|h||\underline{p}^H \underline{r}| \quad \Big| \frac{\partial}{\partial |h|}$$

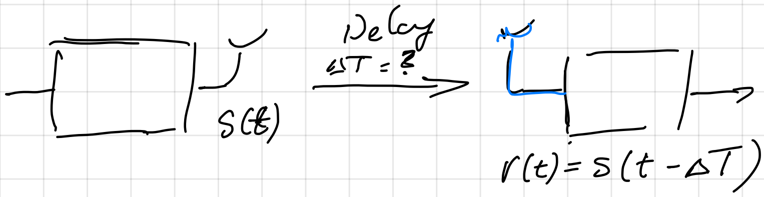
$$= 2|h| N_p - 2|p^H r| \stackrel{!}{=} 0 \Rightarrow |h| = \frac{|\underline{p}^H \underline{r}|}{N_p}$$

with $\hat{\phi}_h = \arg(p^H r)$ $\qquad h = \frac{\underline{p}^H \underline{r}}{N_p}$

Wireless Receivers Lecture 2

Frame Synchronization

Synchronization (Time)



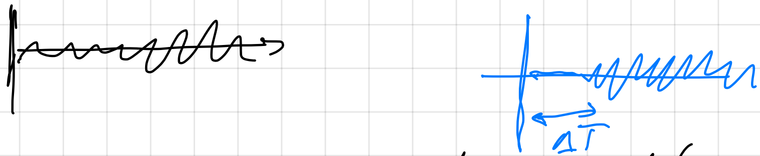
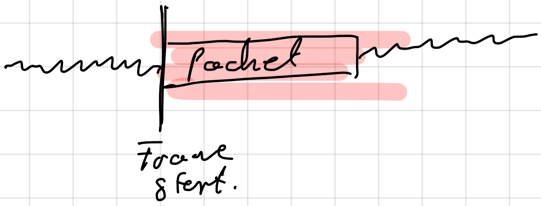
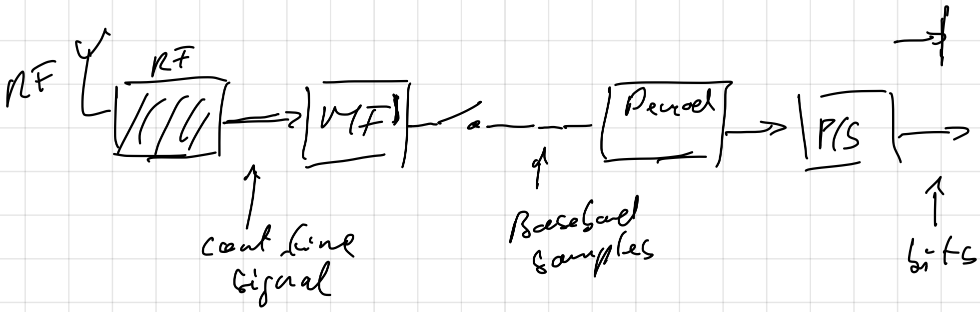


figure out time delay → Est. problem



Detection Problem

Synchronization at diff. Stages

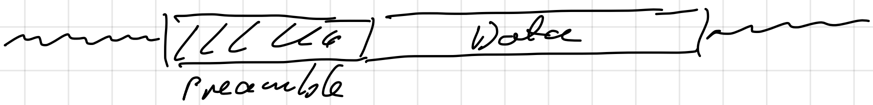


(1) Sync. with subsample resolution ⇒ find right sampling point.

(2) Sync. on a sample-by-sample basis

Today → sample level [(3) Frame start detection

Frame Synchronization with Preamble



Known Seq. for TX and for RX

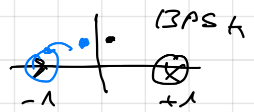
RX Task: looks for Preamble in the RX samples by comparing RX samples to known preamble → Decision

Synchronization Ex. (BINARY → after decod)

Detect a data block in a sequence of bit

TX 0 1 0 1 0 1 x x x x x x t

RX 0 0 1 1 0 1 0 1 0 1 0 1 r



↑ looks for Preamble

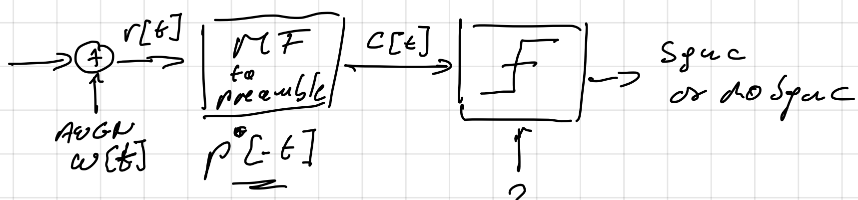
→ Hamming distance based a Hamming dist threshold ≥ 0

Frame start detection on AWGN channel

Compare the RX signal to the preamble

- Matched filter ← compl. conj, time reversed signal we look for
- Correlator

N_p: Preamble length



$$c[t] = \sum_{i=0}^{N_p-1} \tilde{p}[i]\, r[t-i] \qquad \tilde{p}[j] = p[N_p - 1 - j]$$

$$c[t] = \sum_{i=0}^{N_p-1} p^*[i]\, r[t+i]$$

Cross-correlation between RX-signal and the preamble → "measures the similarity"

Preamble design

Noiseless case

BPSK: $p[t] \in \{\pm 1\}$

Data: $d[t] = 0$

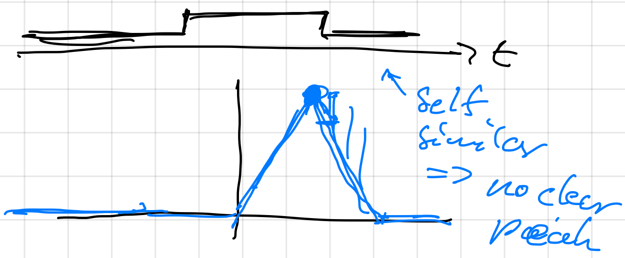
$$r[t] = \begin{cases} d[t] & , t < t_0 \\ p[t-t_0] & , t_0 \le t \le t_0 + N_p - 1 \\ d[t] & , t_0 > N_p - 1 \quad \text{(else)} \end{cases}$$

start of the frame

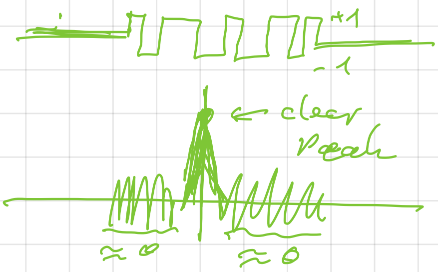
No noise RX signal / TX Preamble is the same

Cross corr → Autocorrelation

1. Const. +1 Preamble



2. Pseudorand ±1



Synchronization

Demod: $p(a=x|r)$ (last time)

$\hookrightarrow p(r|a=x) \rightarrow$ Hypothesis test

Frame Sync 2 Hypotheses $\mathcal{H}_0$: no sync in RX signal

$\mathcal{H}_1$: sync. present in RX signal

$\hookrightarrow$ which is more likely

write RX signal/samples in vector form:

$$\underline{r}[t] = [\,r[t], r[t+1], \dots, r[t+N_p-1]\,]$$

Noise

$\mathcal{H}_0$: no preamble $\underline{r}[t] = \underline{\omega}[t]$

preamble

$\mathcal{H}_1$: with preamble $\underline{r}[t] = h\,\underline{p}[t] + \underline{\omega}[t]$

channel (scalar, compl.)

$$p[t] \longrightarrow \underset{\uparrow h}{\otimes} \longrightarrow \overset{\omega[t]\ \downarrow}{\oplus} \longrightarrow h p[t] + \omega[t]$$

Consider likelihood of $\mathcal{H}_0$ and $\mathcal{H}_1$

$\gamma = 1 \Rightarrow$ Max likelihood

Generalized likelihood test $L_G(\underline{r}) = \dfrac{p(\underline{r}|\mathcal{H}_1)}{p(\underline{r}|\mathcal{H}_0)} > \gamma$

- Dist. of $\underline{r}$ assuming $\mathcal{H}_0$

  - Gaussian noise (N_p dimensional) zero-mean $p(\underline{r}|\mathcal{H}_0) = \dfrac{1}{(\pi\sigma_0^2)^{N_p}} \exp\left(-\dfrac{\|\underline{r}\|^2}{\sigma_0^2}\right)$

  How to get σ_0^2: Estimate from RX signal $\underline{r}$

  $$\sigma_0^2 = \|\underline{r}\|^2 \cdot \left(\frac{1}{N_p}\right) \leftarrow \text{per sample}$$

- Distribution of $\underline{r}$ assuming $\mathcal{H}_1$

  - Gaussian with mean of $\underline{p} \leftarrow$ Preamble

$$p(\underline{r}|\mathcal{H}_1) = \frac{1}{(\pi\sigma_1^2)^{N_p}} \exp\left(-\frac{\|\underline{r} - h\underline{p}\|^2}{\sigma_1^2}\right)$$

$\boxed{\underline{p}^H \cdot \underline{p} = N_p}$ $p[t] \in \{\pm 1\}$

  - Get σ_1^2 from Rx-signal: $\sigma_1^2 = \frac{1}{N_p}\|\underline{r}\|^2 - |\underline{h}|^2 \leftarrow ?$

How to get h or actually $|h|$?

p^H = Hermitian transpose

$\begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_N \end{bmatrix} = p$

$p^H = [p_1^*, p_2^*, \dots p_N^*]$

$\|p\| = p^H p$

↳ Estimate the most likely h from rx signal

$$\hat{h} = \arg\max_h p(\underline{r} | \mathcal{H}_1, h) \Rightarrow \text{max of an exp.}$$

↳ minimize $\|r - hp\|^2$

$$\arg\min_h \underbrace{\|\underline{r} - hp\|^2}_{Z}$$

$$Z = \cancel{\|\underline{r}\|^2} - \underline{r}^H p h - h p^H r + |h|^2 \underbrace{\|p\|^2}_{N_p}$$

write $h = |h| e^{j\phi_h}$

$$L = -2\,\mathrm{Re}\{p^H \underline{r}\, |h|\, e^{-j\phi_h}\} + |h|^2 N_p$$

make this real-valued $\phi_h = \arg(p^H r)$

$$L = |h|^2 N_p - 2|h|\,|p^H \underline{r}| \quad \Big| \frac{\partial}{\partial |h|}$$

$$2|h| N_p - 2|p^H r| \overset{!}{=} 0 \longrightarrow |h| = \frac{|p^H r|}{N_p}$$

with $\phi = \arg(p^H r)$ $\longrightarrow$ $\hat{h} = \frac{p^H r}{N_p}$

↳ Now go back to finding $\sigma_1^2 = \frac{1}{N_p}\|\underline{r}\|^2 - |h|^2$

$$\sigma_1^2 = \frac{1}{N_p}\left(\|r\|^2 - \frac{|p^H r|^2}{N_p}\right)$$

Generalized likelihood test

$$L(\underline{r}) = \frac{P(\underline{r}|\mathcal{H}_1)}{P(\underline{r}|\mathcal{H}_0)} = \left(\frac{\frac{\pi}{N_p}|\underline{r}|^2}{\frac{\pi}{N_p}\left(\|r\|^2 - \frac{|p^H r|^2}{N_p}\right)}\right)^{N_p} = \left(\frac{1}{1 - \frac{|p^H r|^2}{|r|^2 N_p}}\right)^{N_p} \gtrless \gamma$$

This is relevant

Adjust γ so that i can consider

$$\Longleftrightarrow \frac{|p^H \underline{r}|^2}{|\underline{r}|^2} \gtrless \gamma''$$

Squared magnitude of the MF output

due to h

scaled MF output.